MAKING ORDER IN CHAOS

Tomaz Prosen, FMF, UL

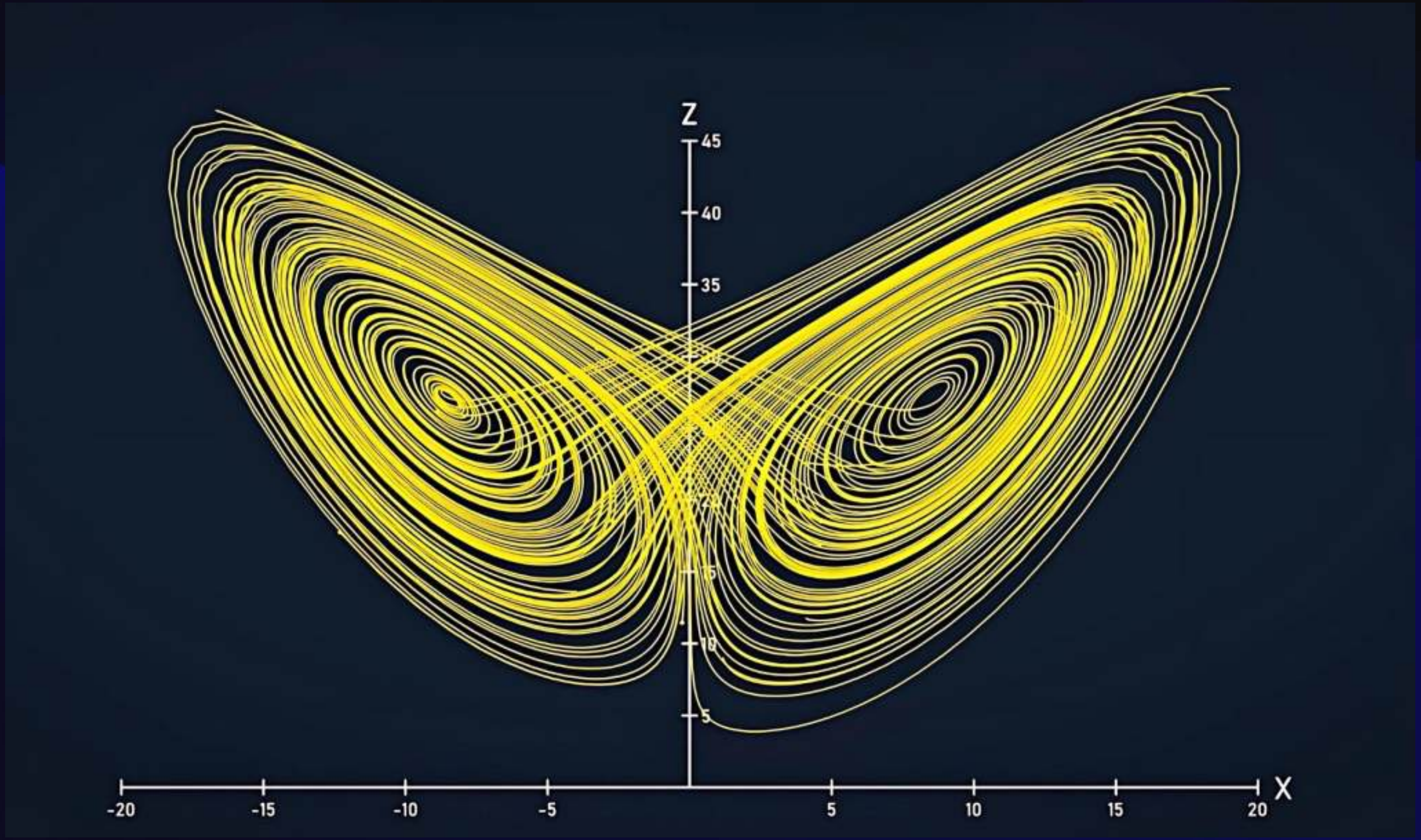
What is (classical) chaos?

Are there universal physical concepts
and principles which allow to precisely define
(and understand!) chaos?

Disclaimer: Deterministic Chaos is not "Chaos" !

POPULAR NOTION OF CHAOS: THE BUTTERFLY EFFECT

Sensitive dependece on initial conditions

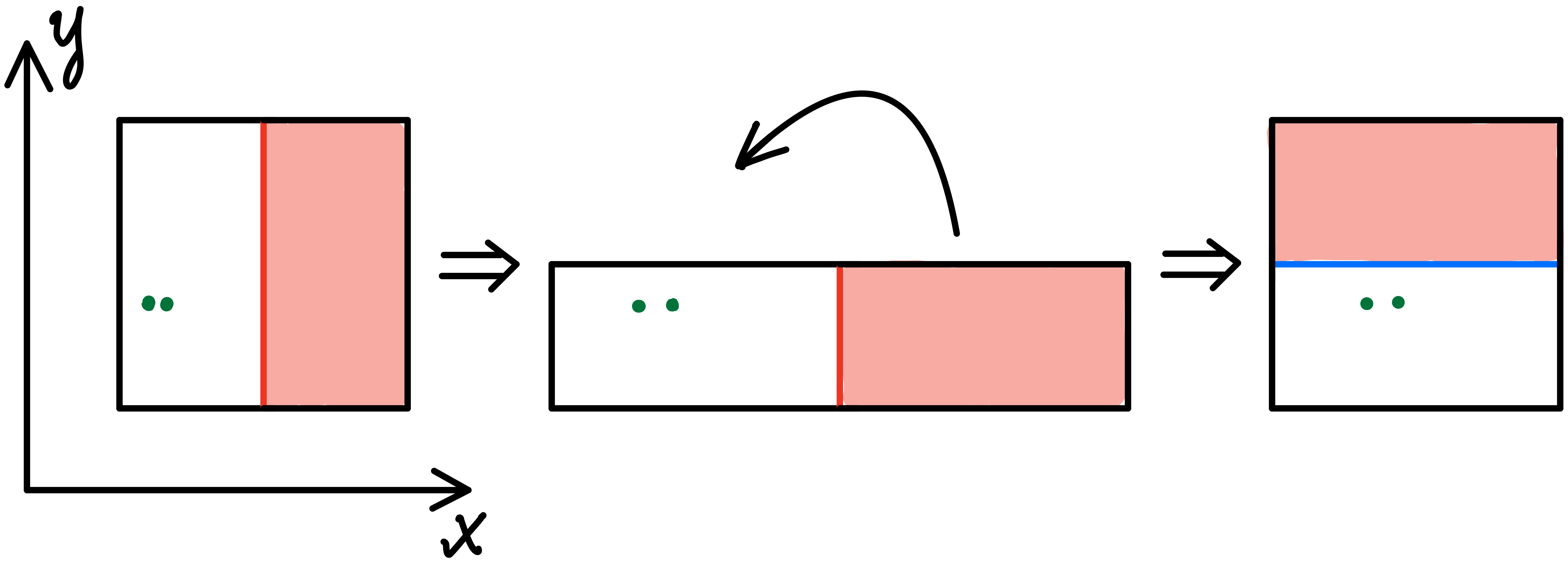


so-called
Lorenz attractor

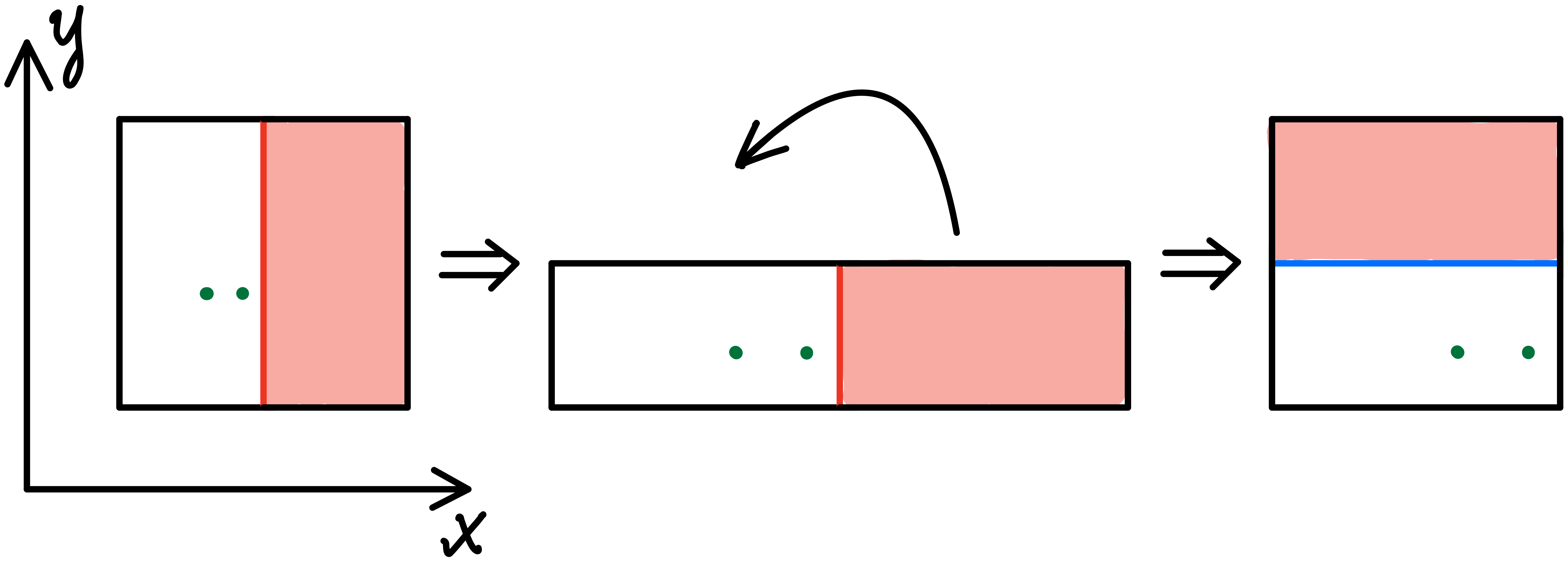
Analitically tractable models of CHAOS

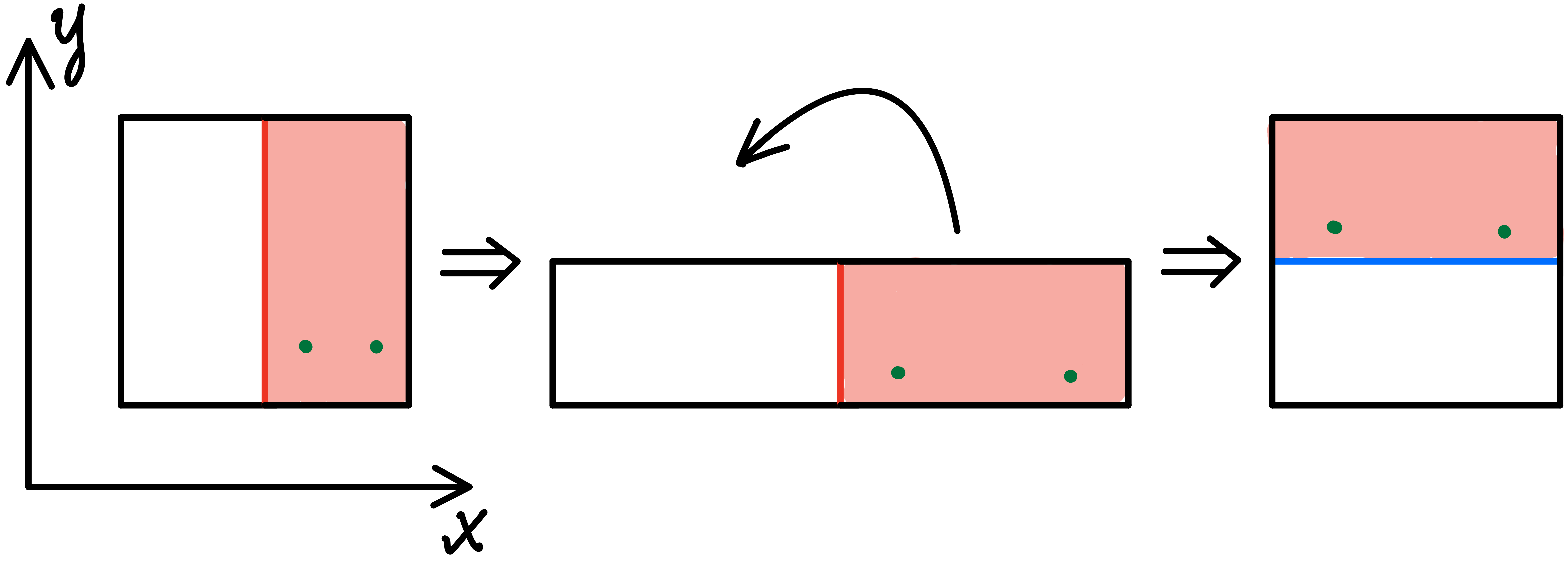
Example 1

BAKER MAP

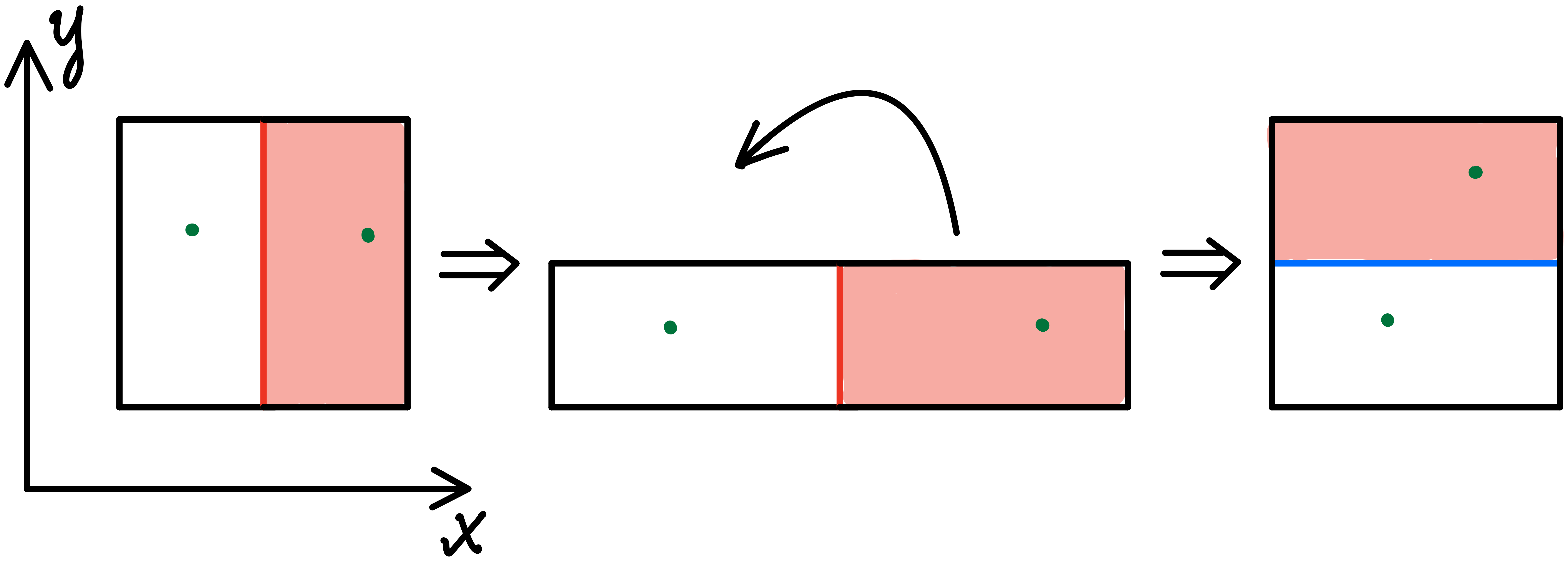


y
x


y
x



y
x



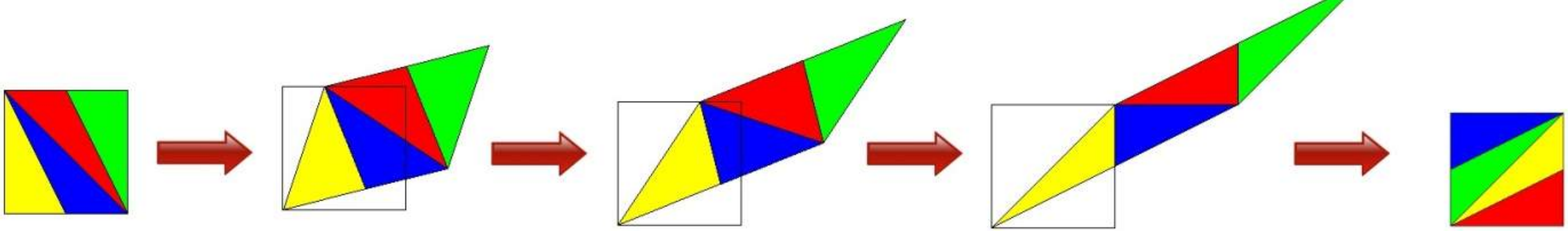
x coordinate is multiplied by 2 in each step (with the integer part forgotten).

The system amplifies one bit per time step of “error” in initial condition.

Entropy (or information) is produced per unit time!

Example 2

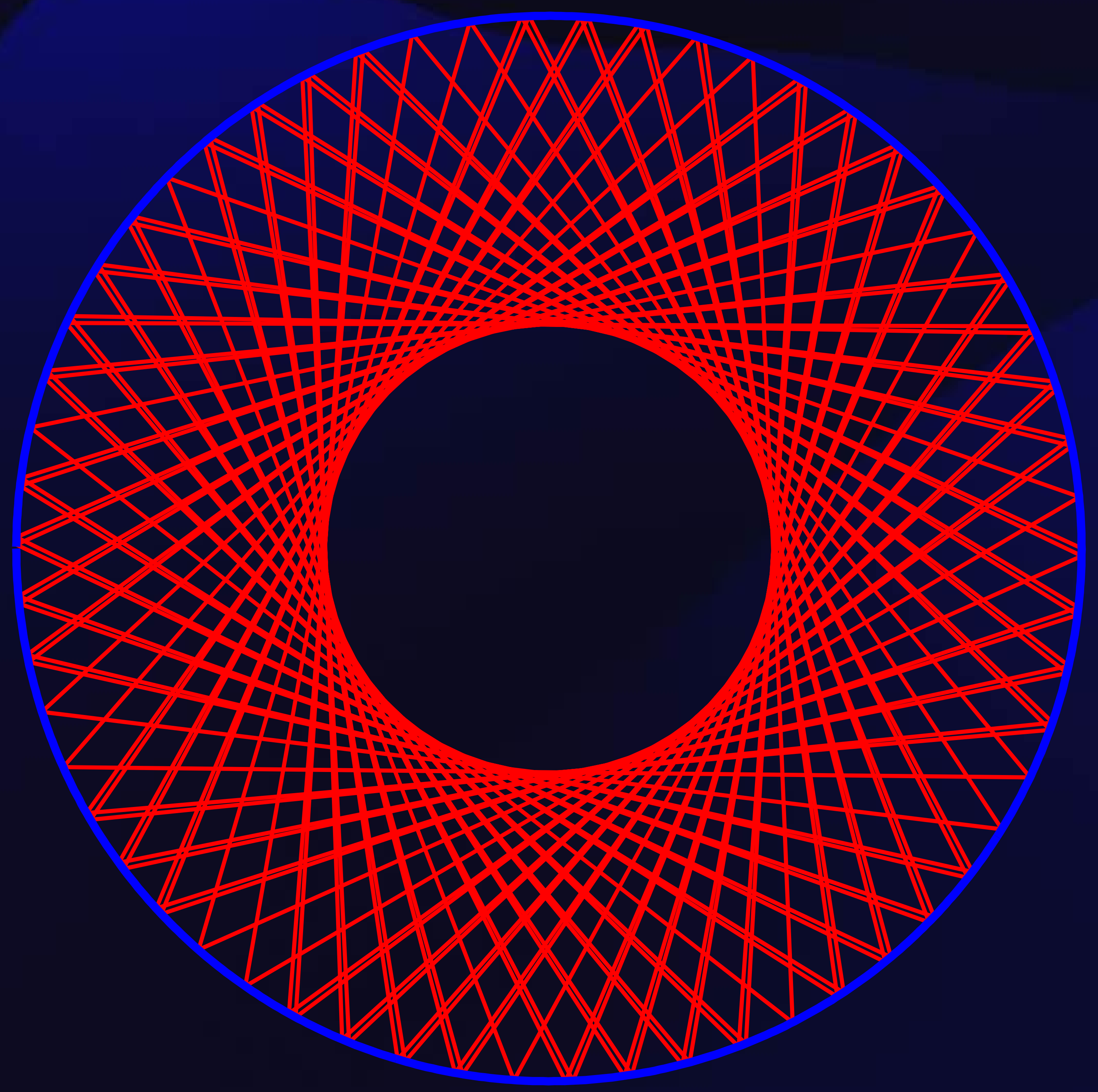
Arnold Cat Map

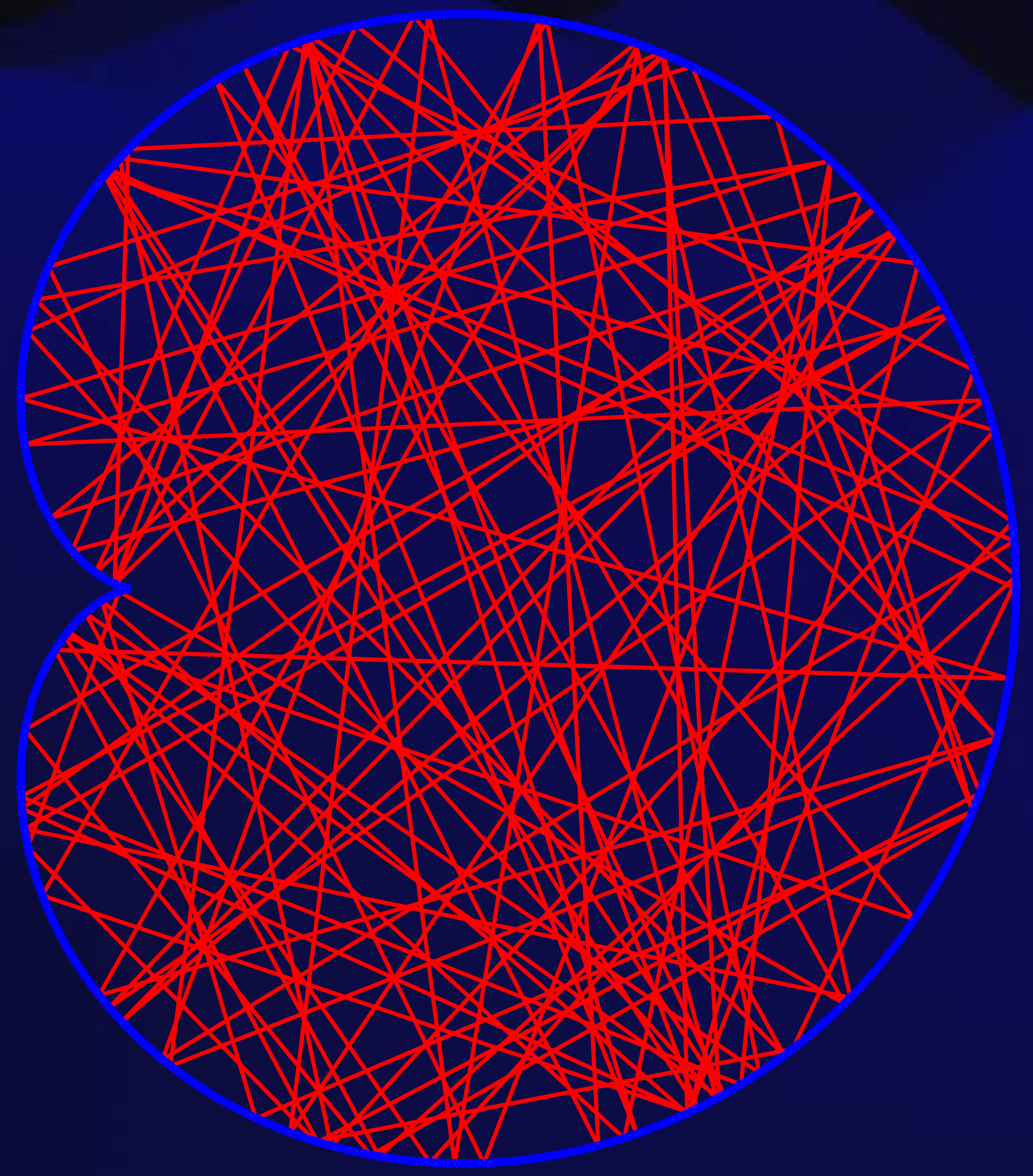


Forgetting the intitial state: Dynamical mixing

Example 3

Order vs Chaos in Billiards



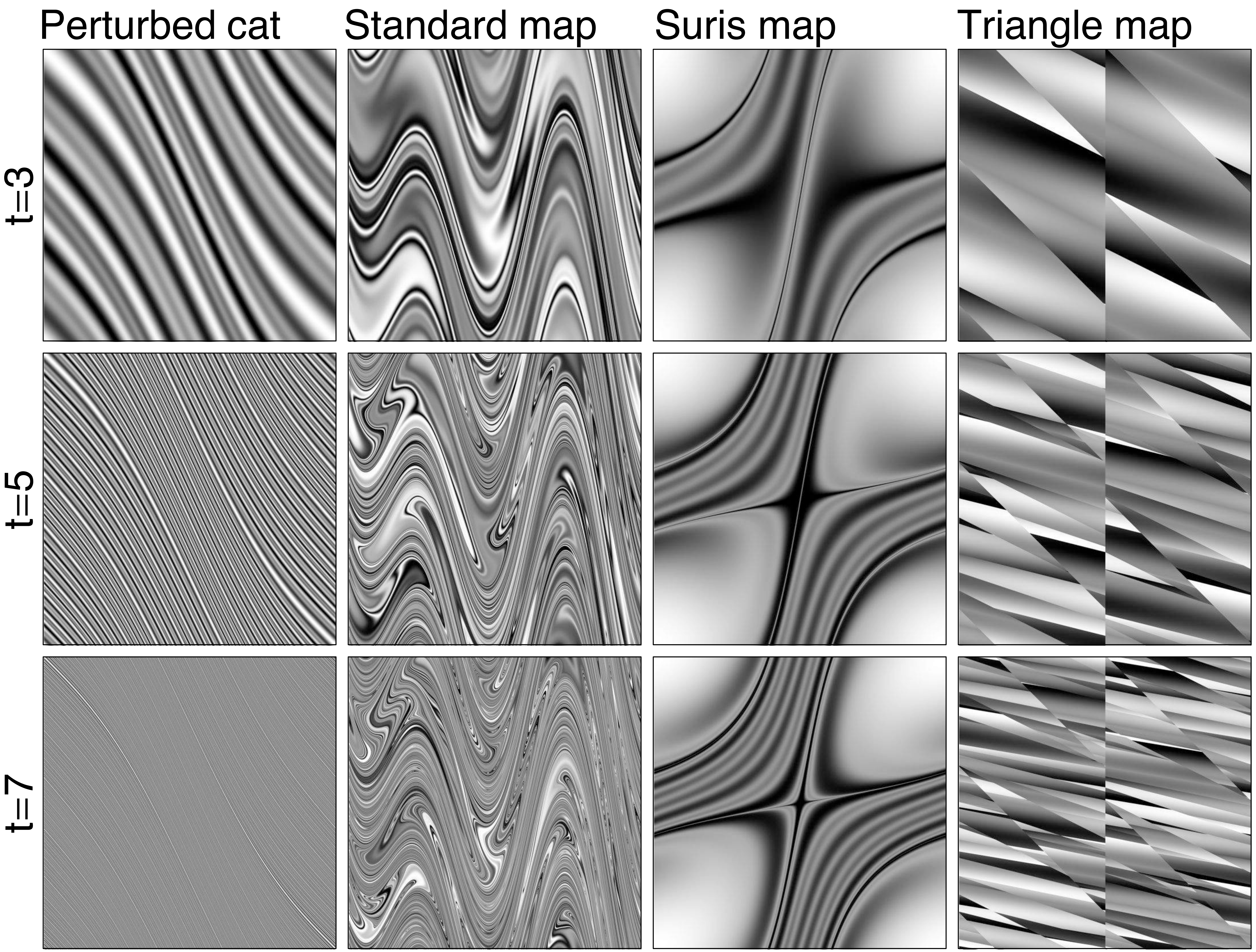


Another feature of chaos: Difficulty of “phase-space” image compression

(Separability Entropy [Prosen 2011])

Comparing four simple dynamical maps:

$$(q_{t+1}, p_{t+1}) = F(q_t, p_t)$$



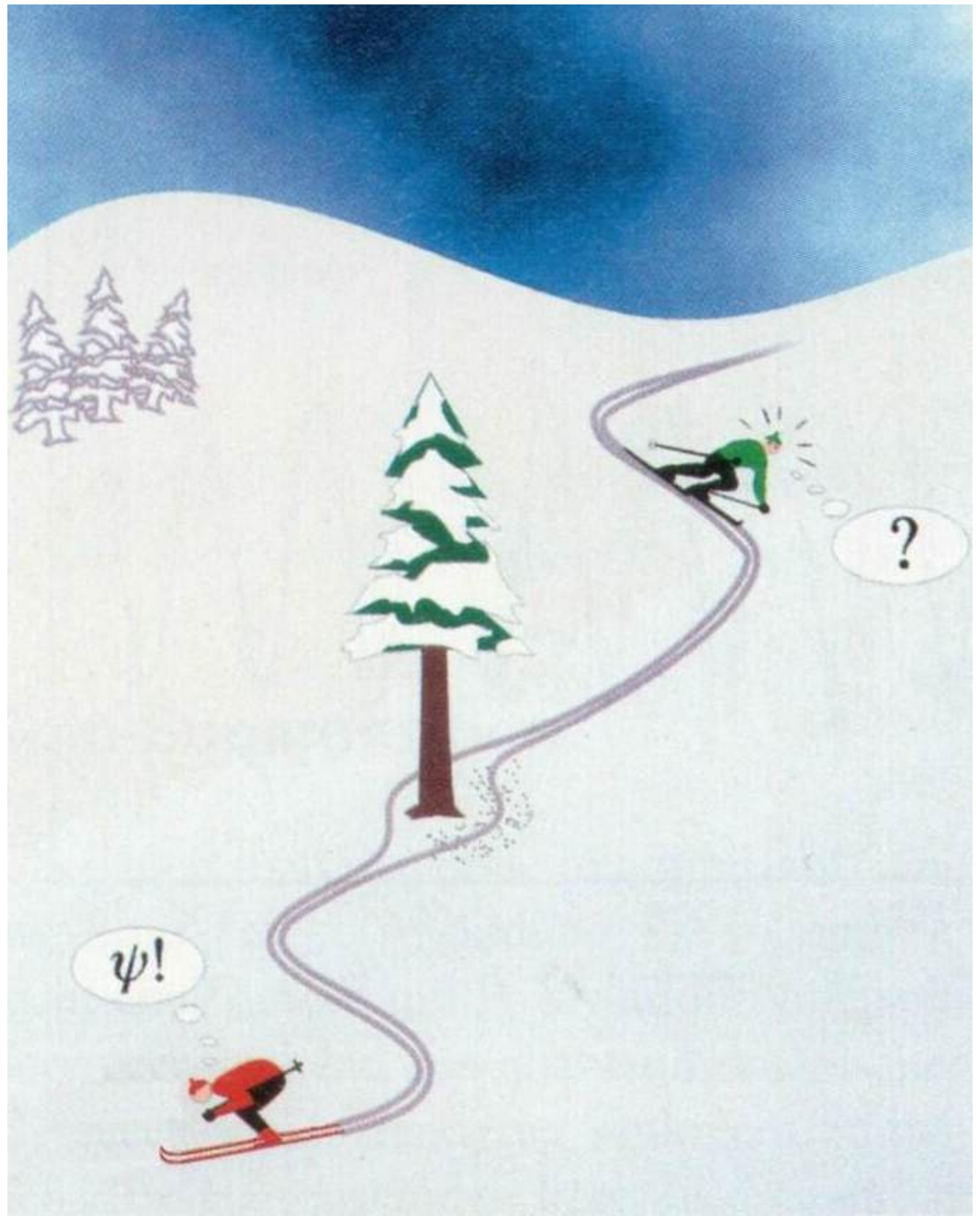
Summary: defining features of classical Chaos

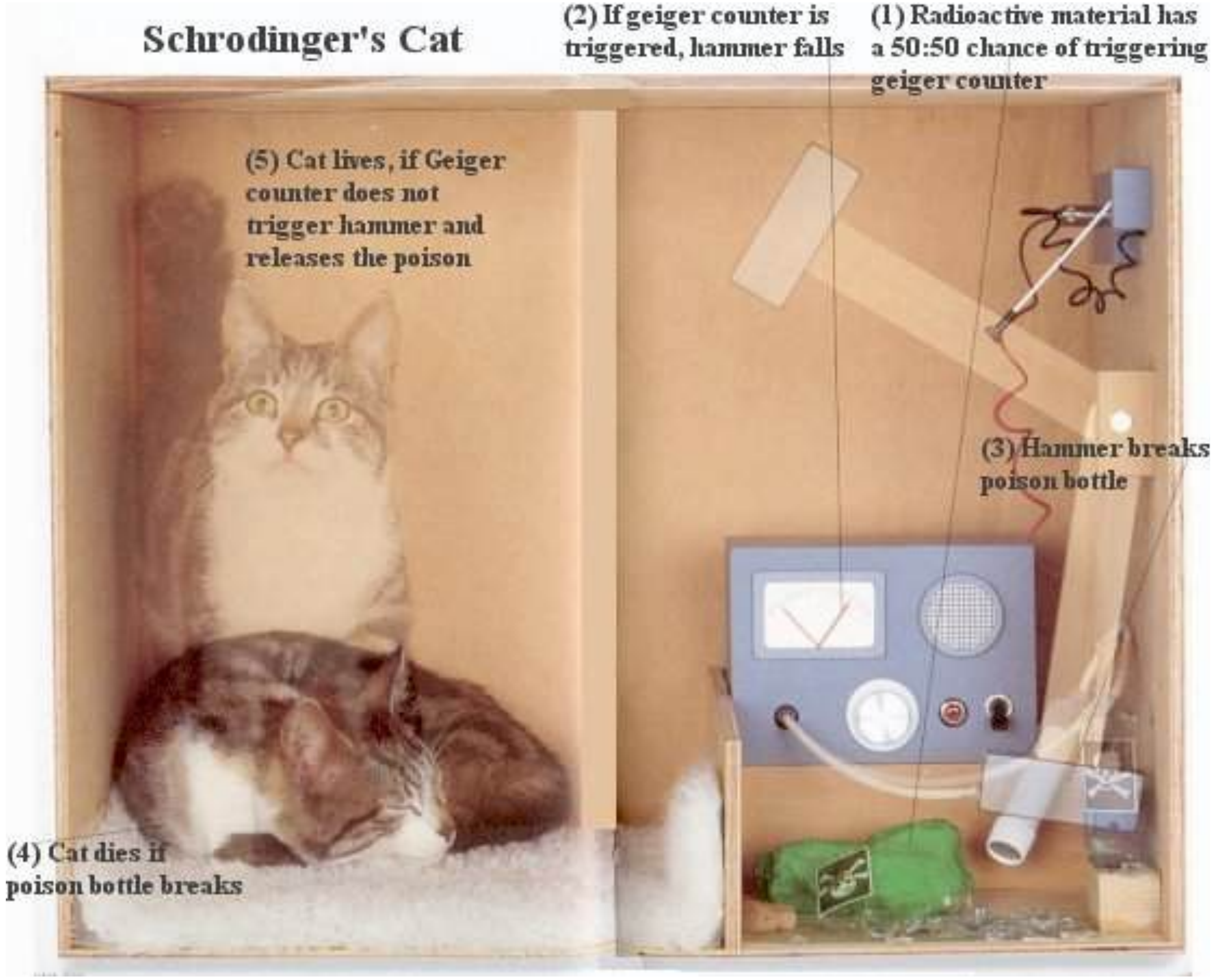
- Exponential sensitivity to initial conditions
- Equivalent: Dynamical production of entropy/information
- Forgetting memory of initial state
- Equivalent: Phase-space structures need finer and finer resolution to be resolved to longer and longer times (phase-space corse-graining needed for entropy growth and 2nd law)

What about quantum chaos?

Are there universal physical features of quantized chaotic systems?

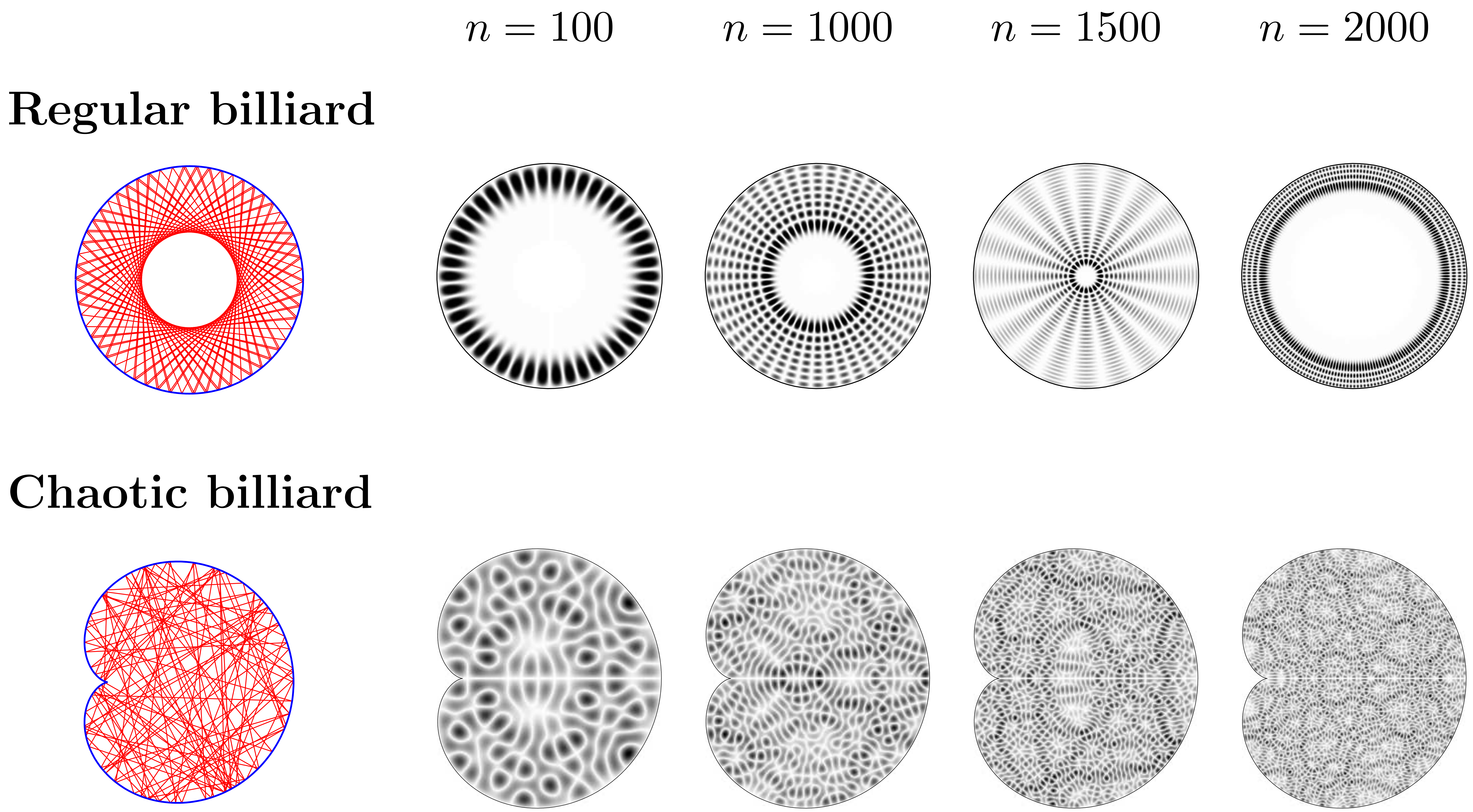
Is there a meaningful definition of quantum chaos in the absence of classical correspondence?





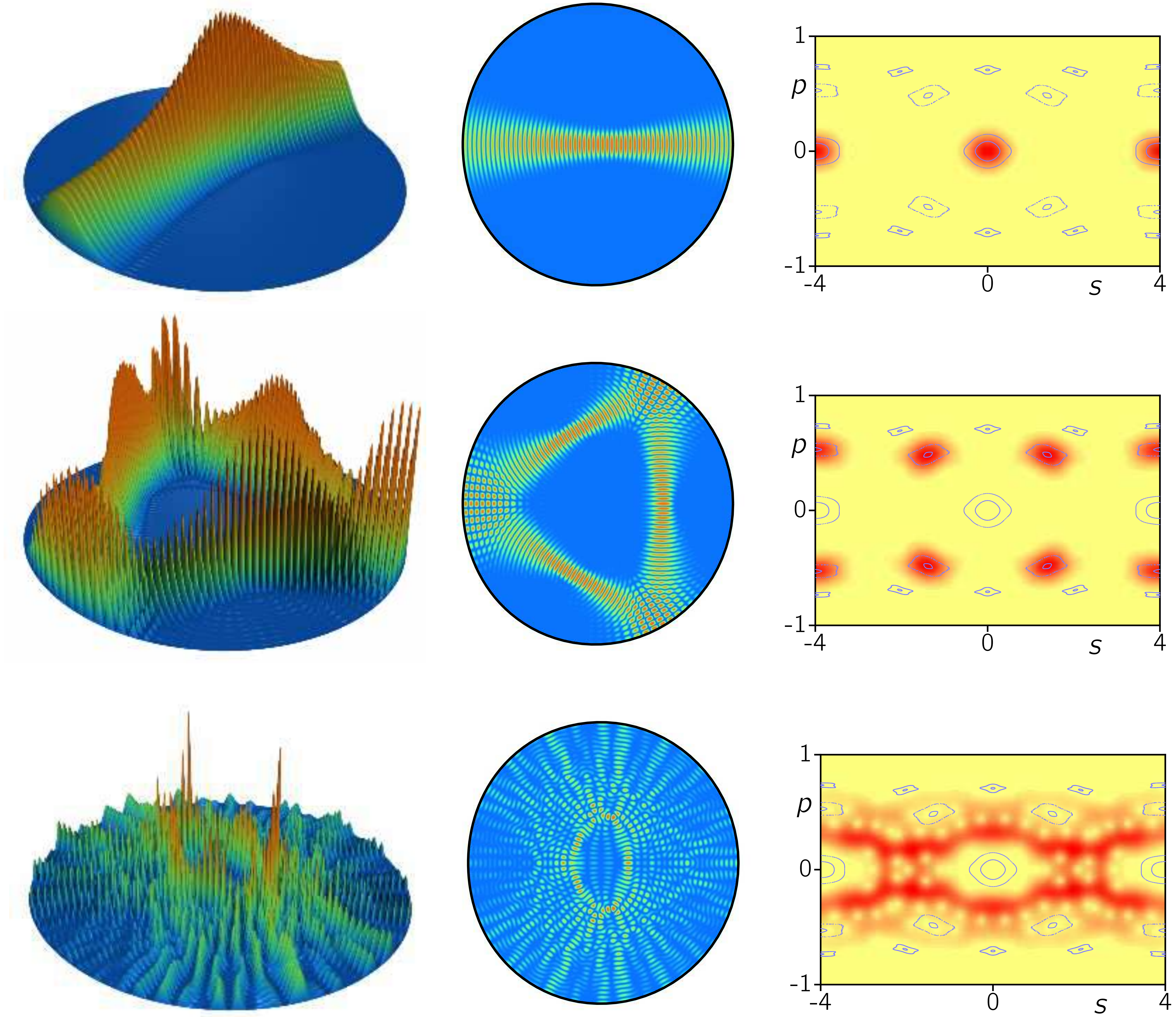
$$|\Psi\rangle = |\text{dead}\rangle + |\text{alive}\rangle$$

Wave chaos



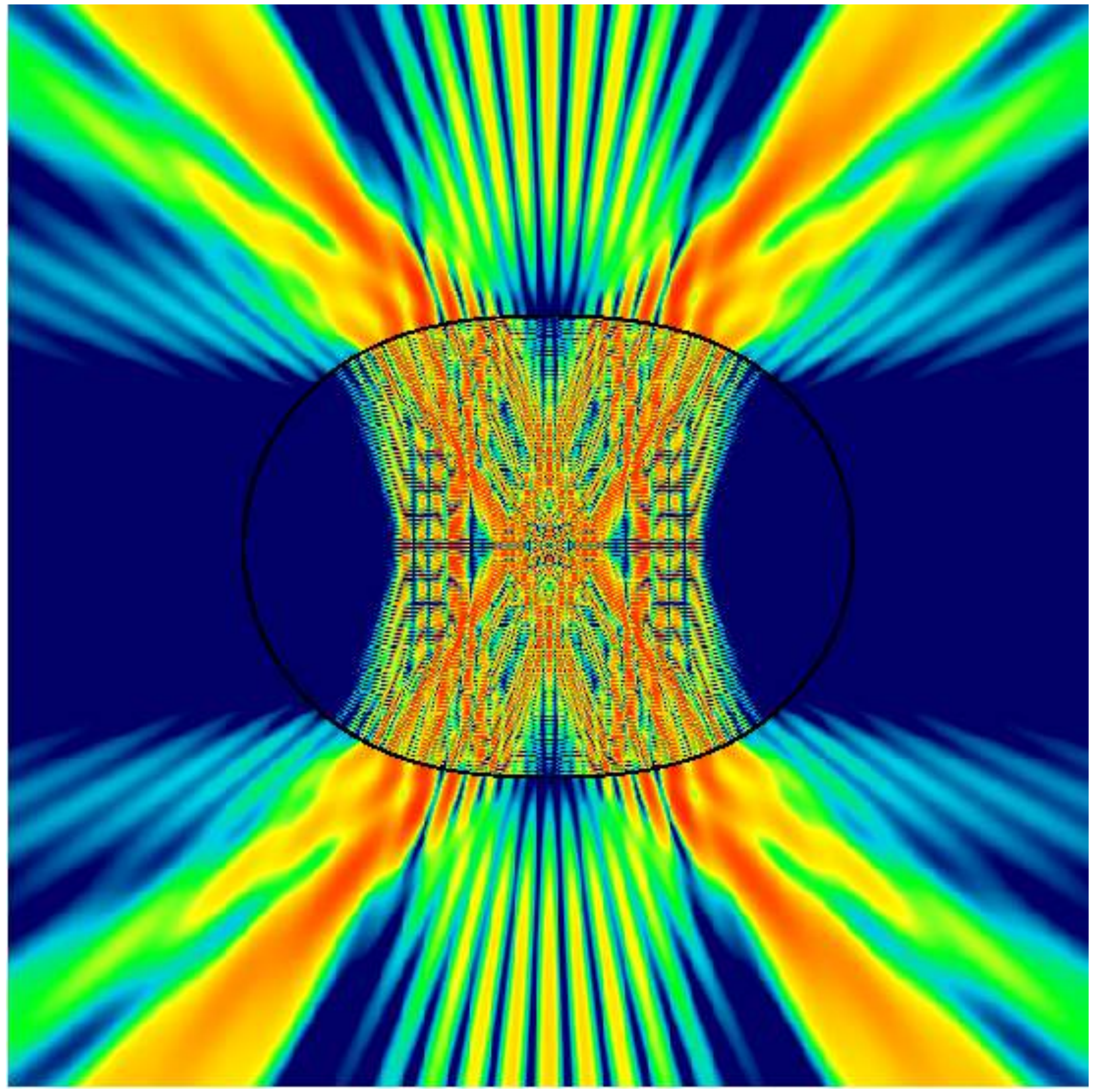
from [Baecker 2007]

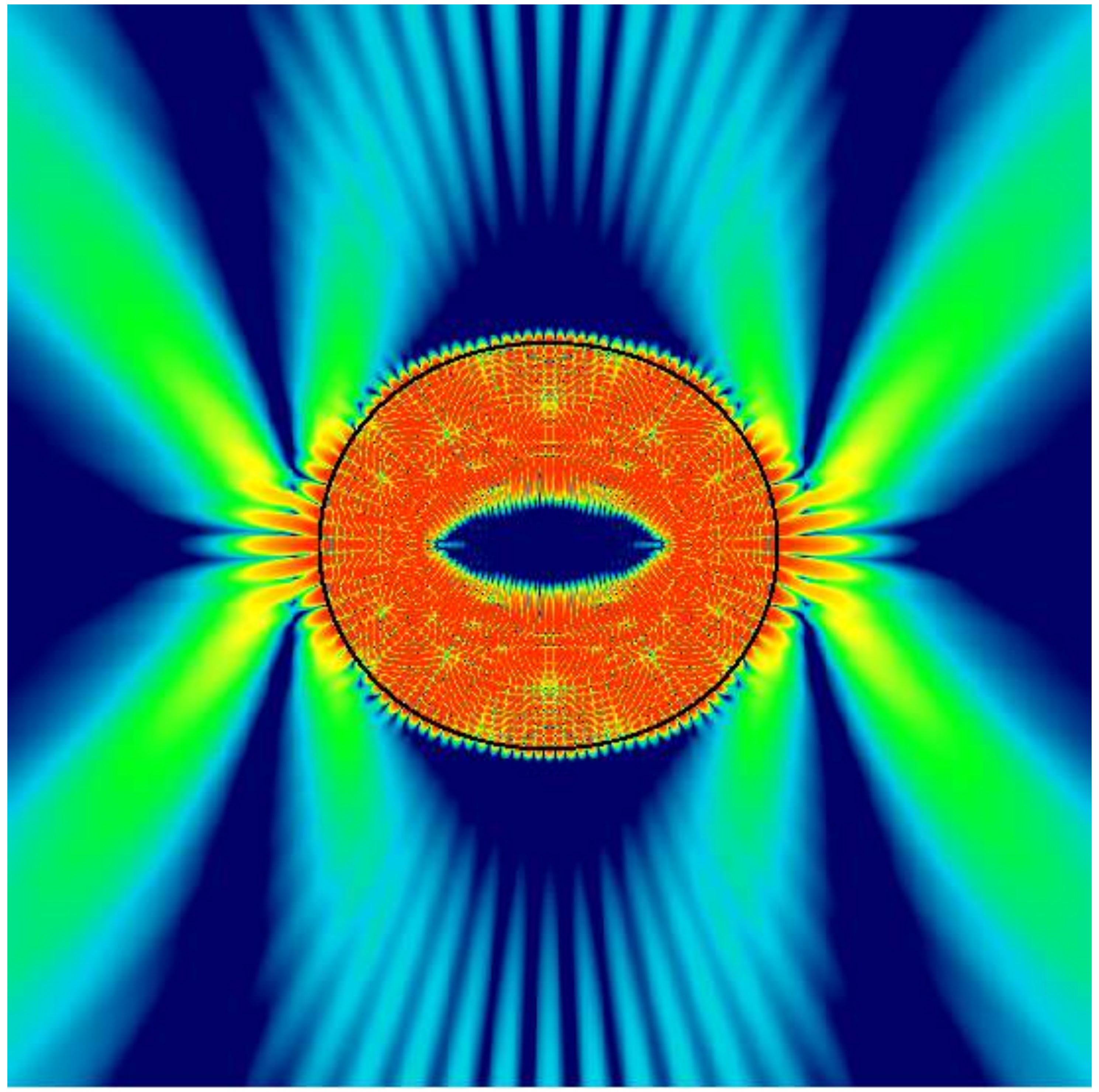
Systems in mixed regime: coexistence of regular and chaotic states



from [Baecker 2007]

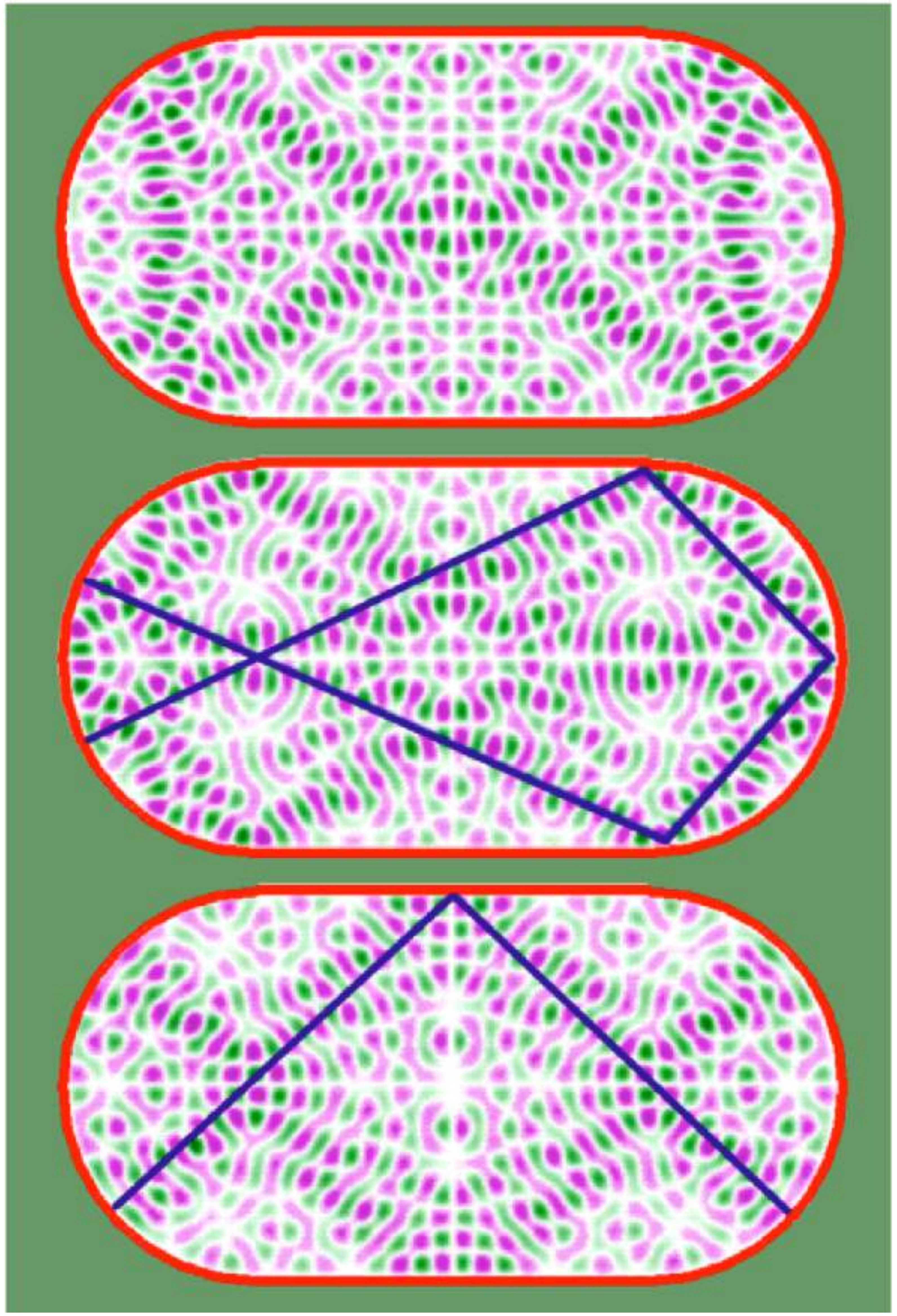
Open Billiards
Application to micro-lasers with directed emission





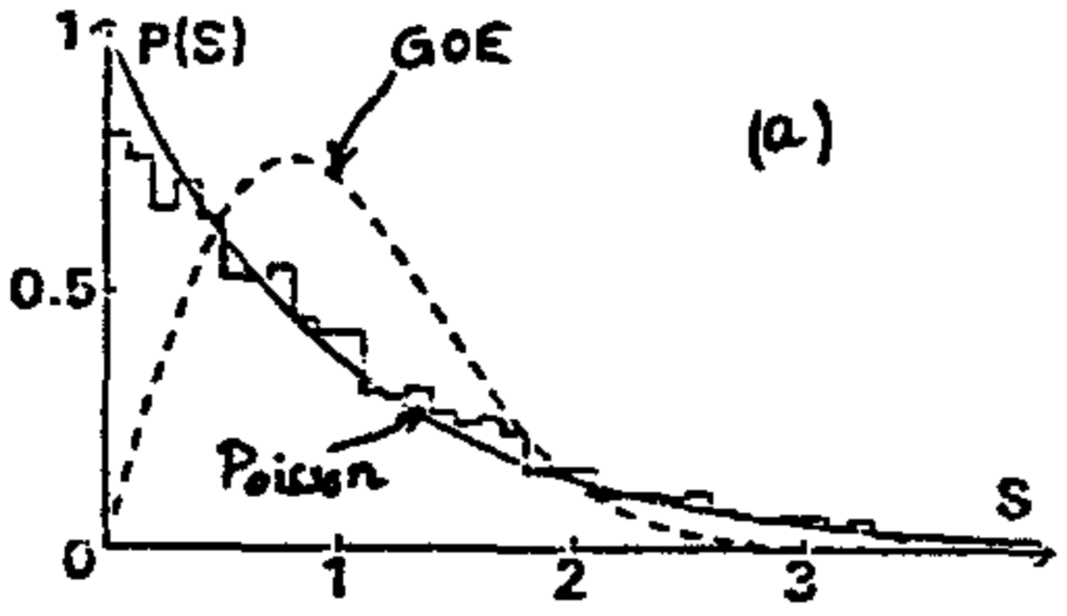
from [Noeckl: https://pages.uoregon.edu/noeckel/]

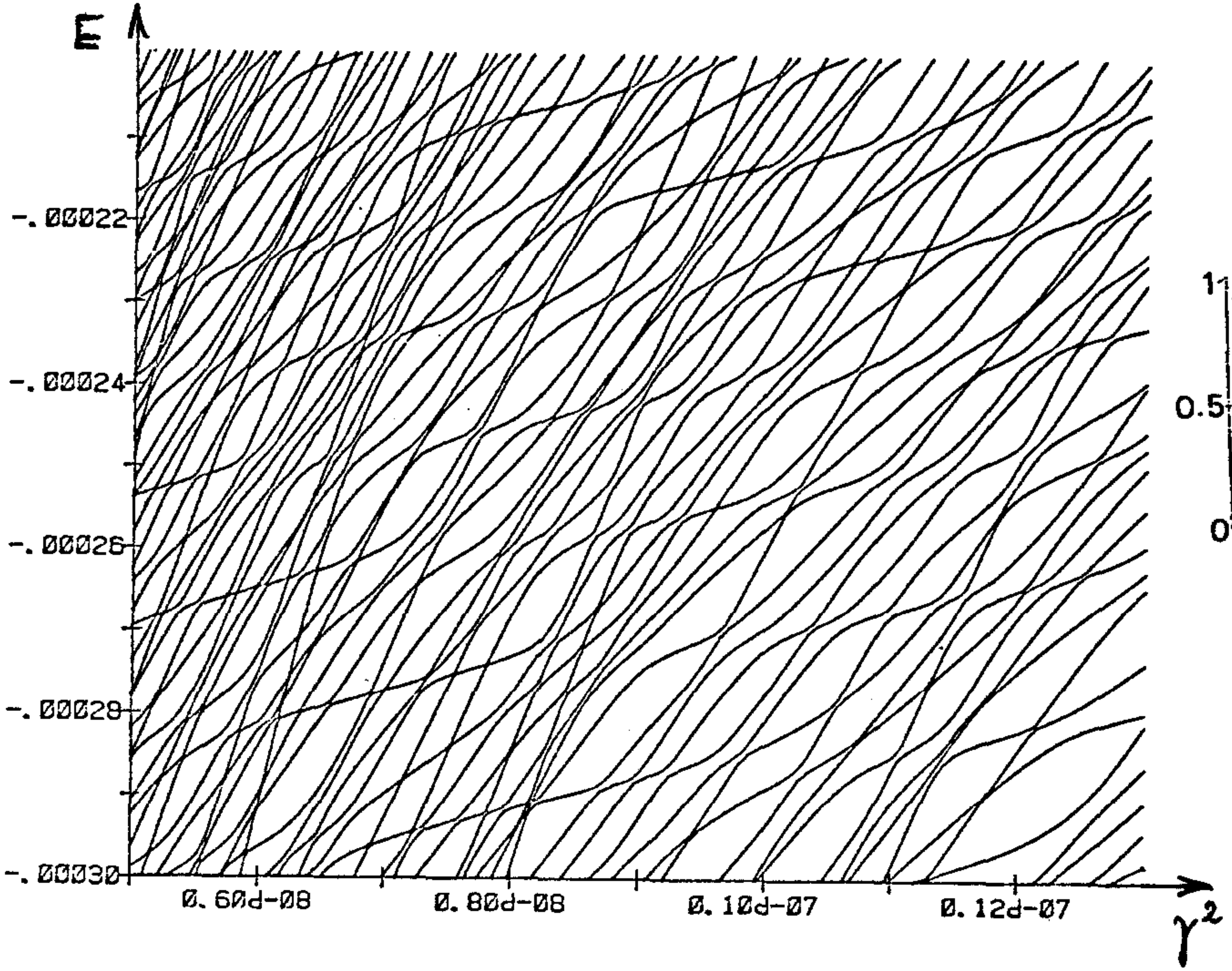
Quantum Scars
- outlaws of quantum chaos -

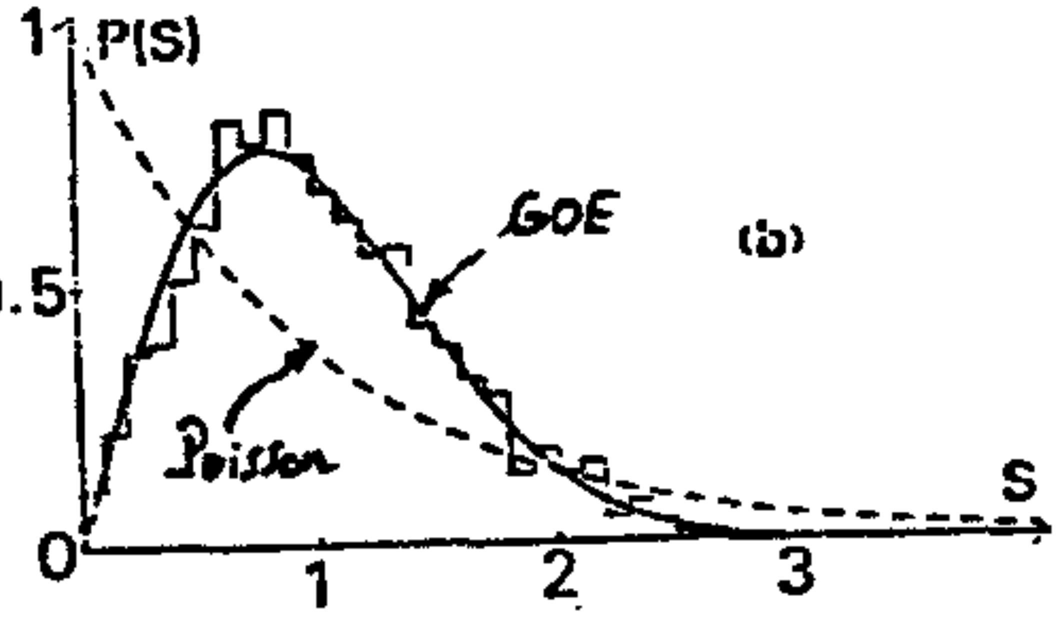


from wikipedia

Energy level flow in Hydrogen atom in strong magnetic field: from order to chaos





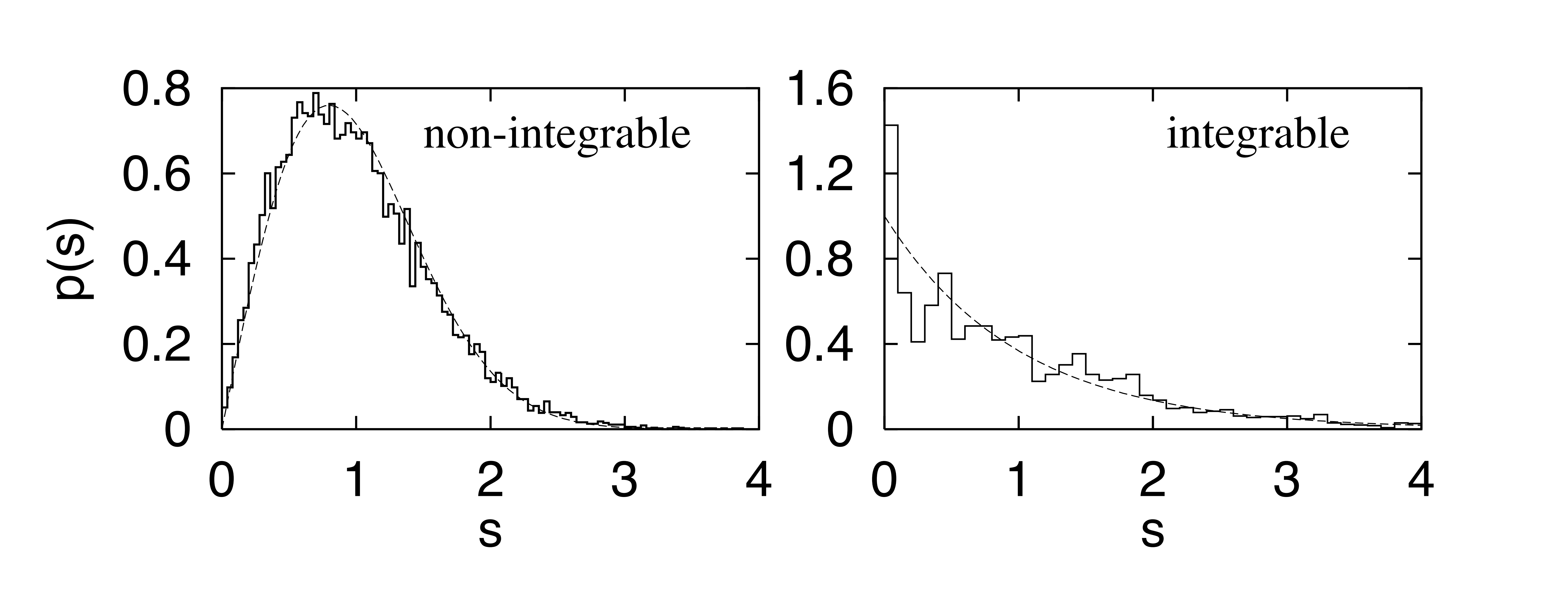


from [Delande and Gay, 1986]

Quantum Chaos Conjecture
(Bohigas et al. 1984, Casati et al. 1980)

Energy spectra of quantum systems whose classical limit is chaotic can be statistically described by random Hamiltonians - unstructured random Hermitian matrices with i.i.d. entries. *Heuristic proof: [Mueller et al. 2005] - for a mathematical proof we are still waiting..*

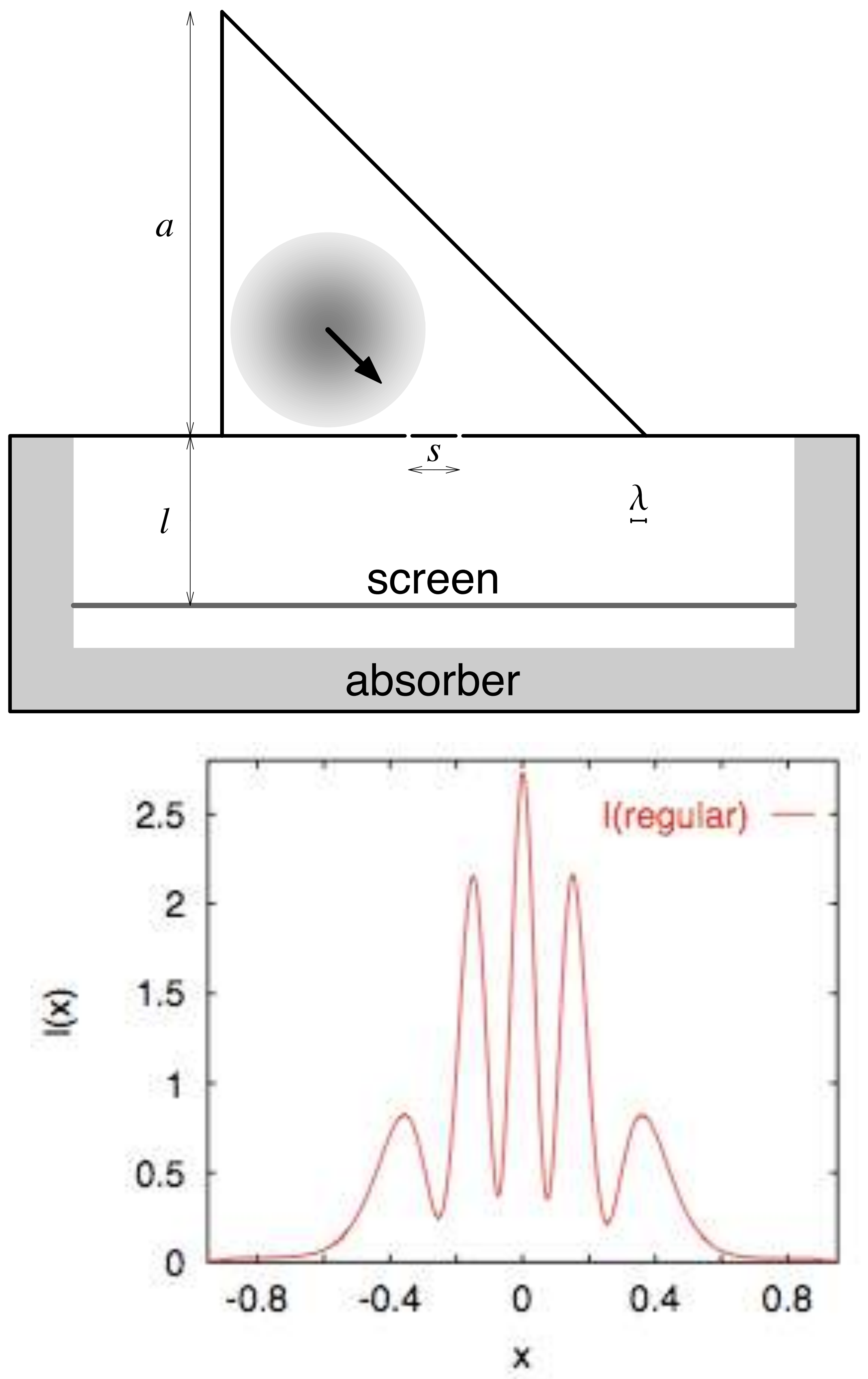
The "reverse": Berry-Tabor (1977) conjecture: Energy spectra of integrable systems are statistically described by a Poisson point process on a line

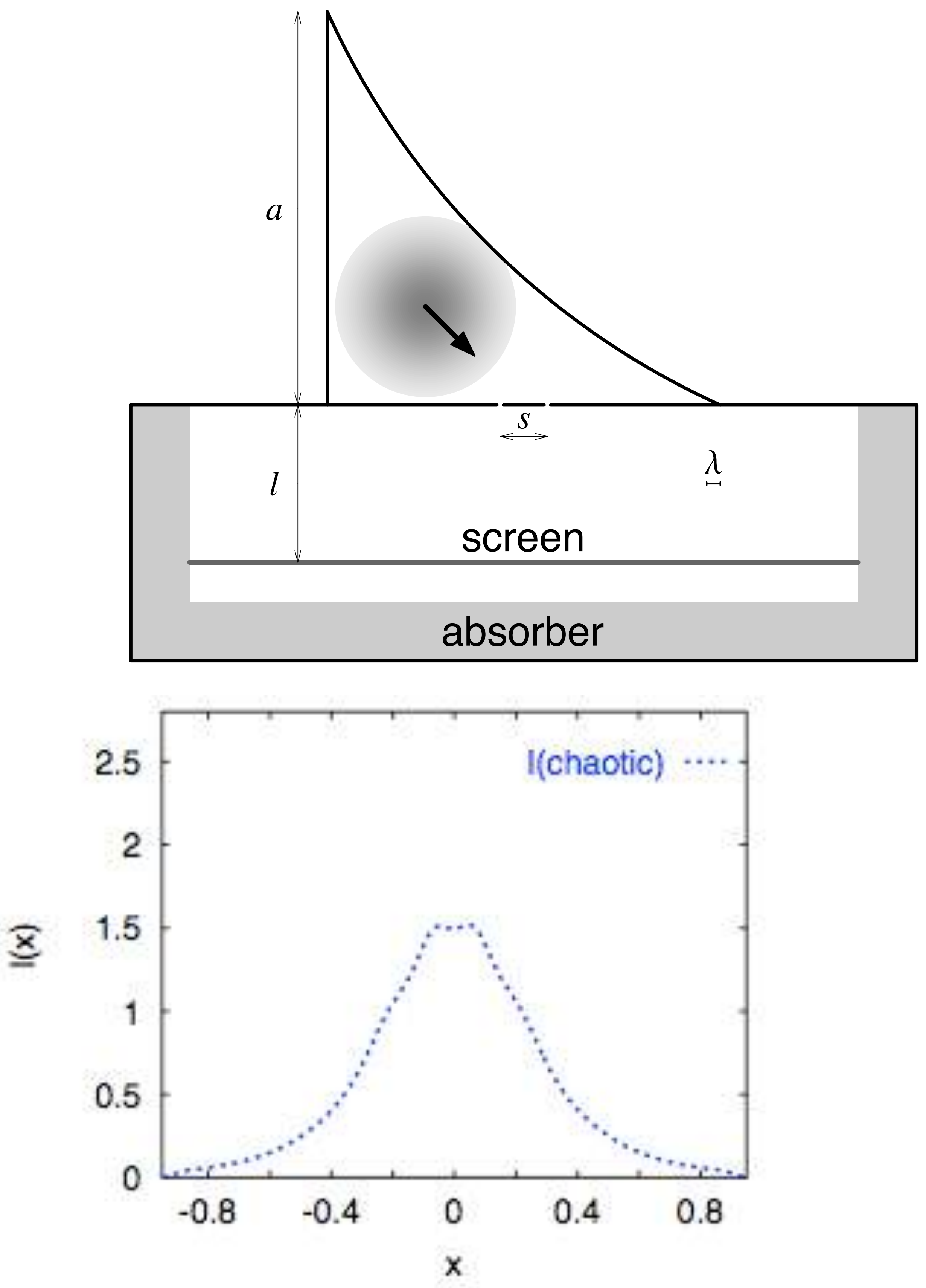


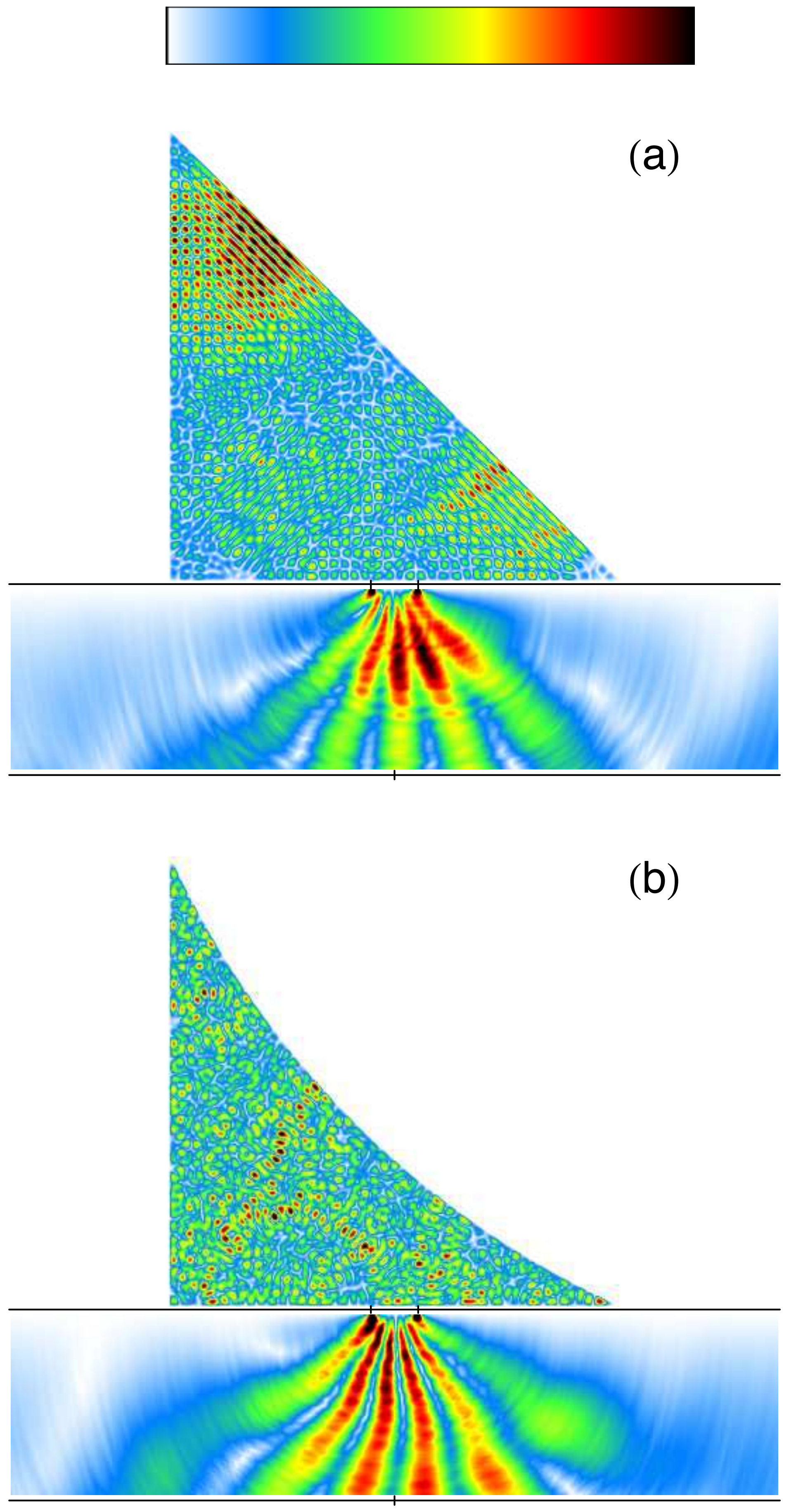
More chaos, more ordered spectrum!

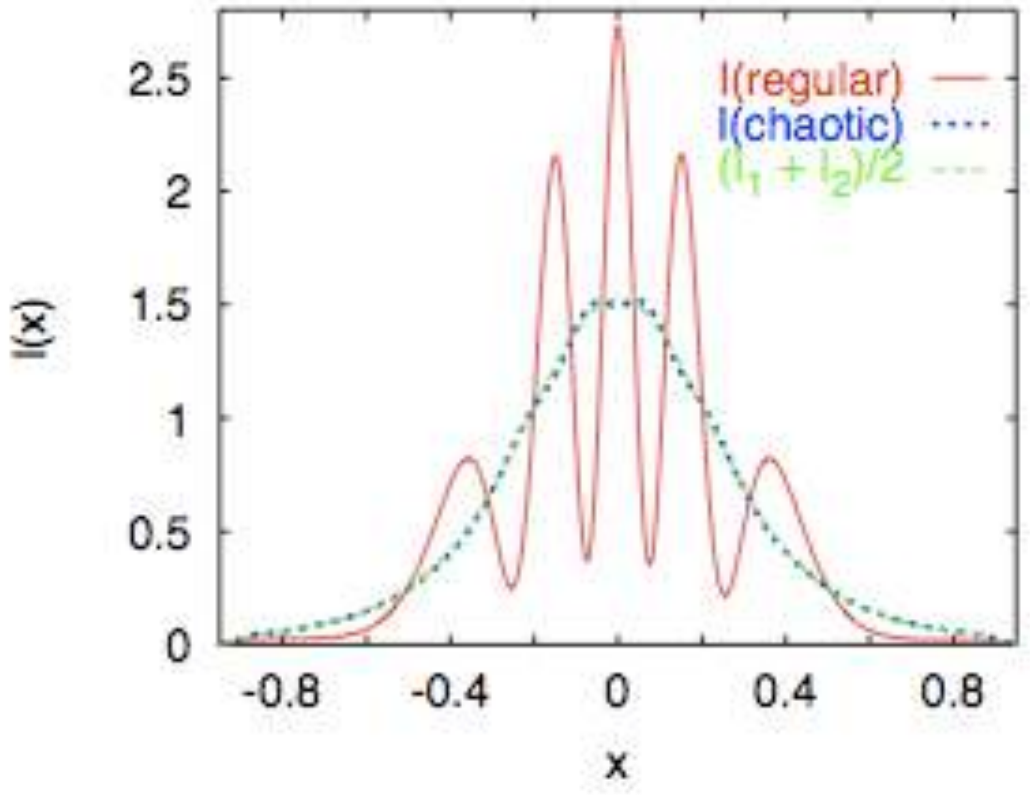
Quantum Chaos and double-slit (numerical) experiment

[Casati and TP, PRA 2005]









modern development:

Many-Body Quantum Chaos

(with or without(!) classical counterpart)

Examples:
-quantum spin (qubit) chains
-models of black hole (SYK)

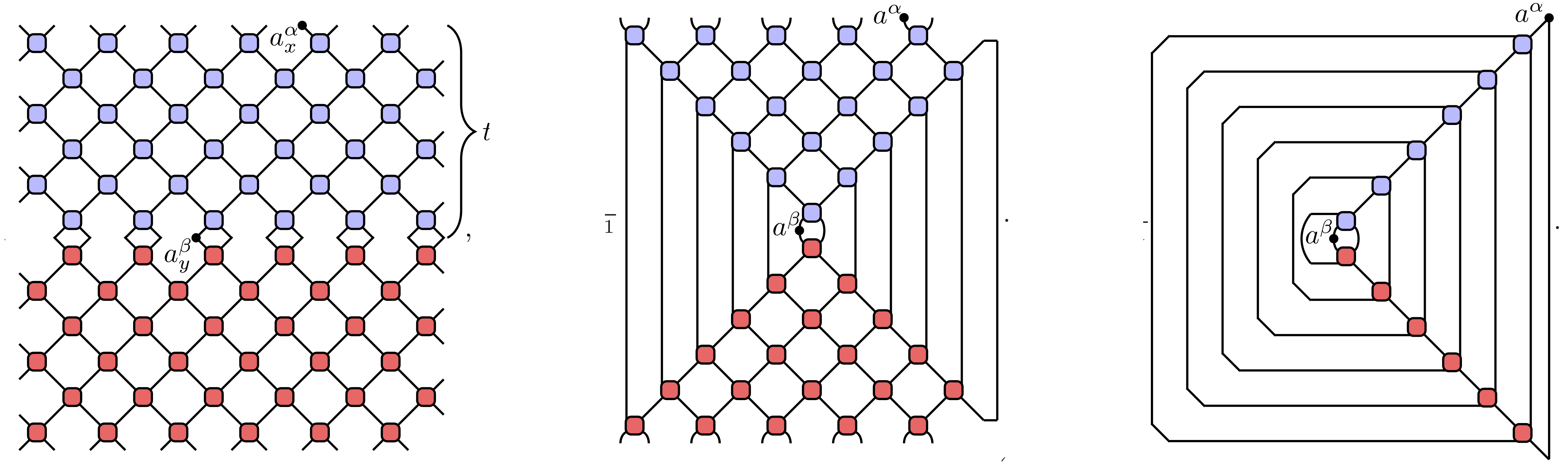
Chaotic quantum spin chains are perfect testbed examples for quantum computers

Google’s latest quantum chip (Willow)

Dual Unitary Quantum Circuits

Example of exactly solvable model of many-body quantum chaos

[Bertini, Kos, TP, 2019]



Ergodic hierarcy of dual unitary systems

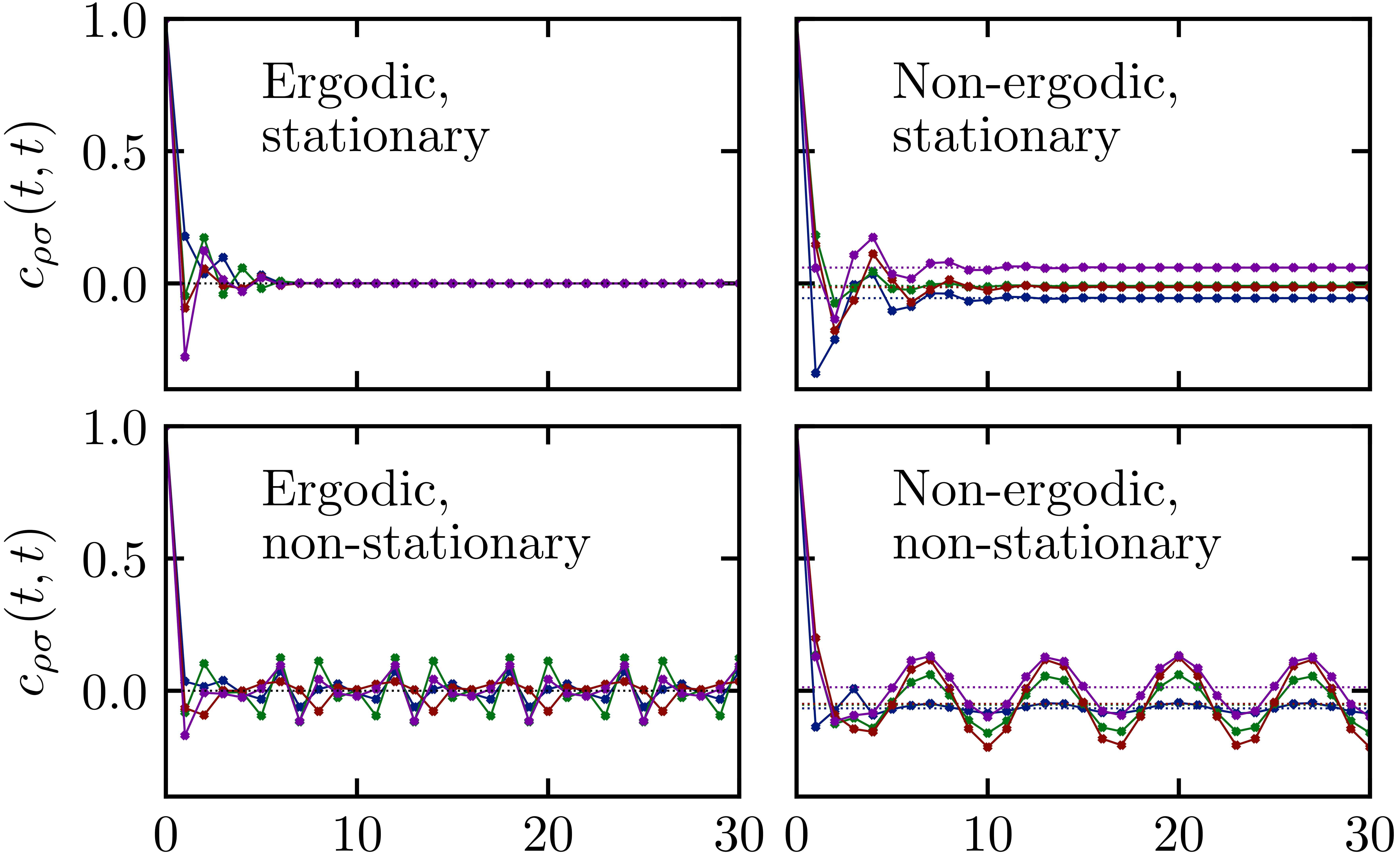
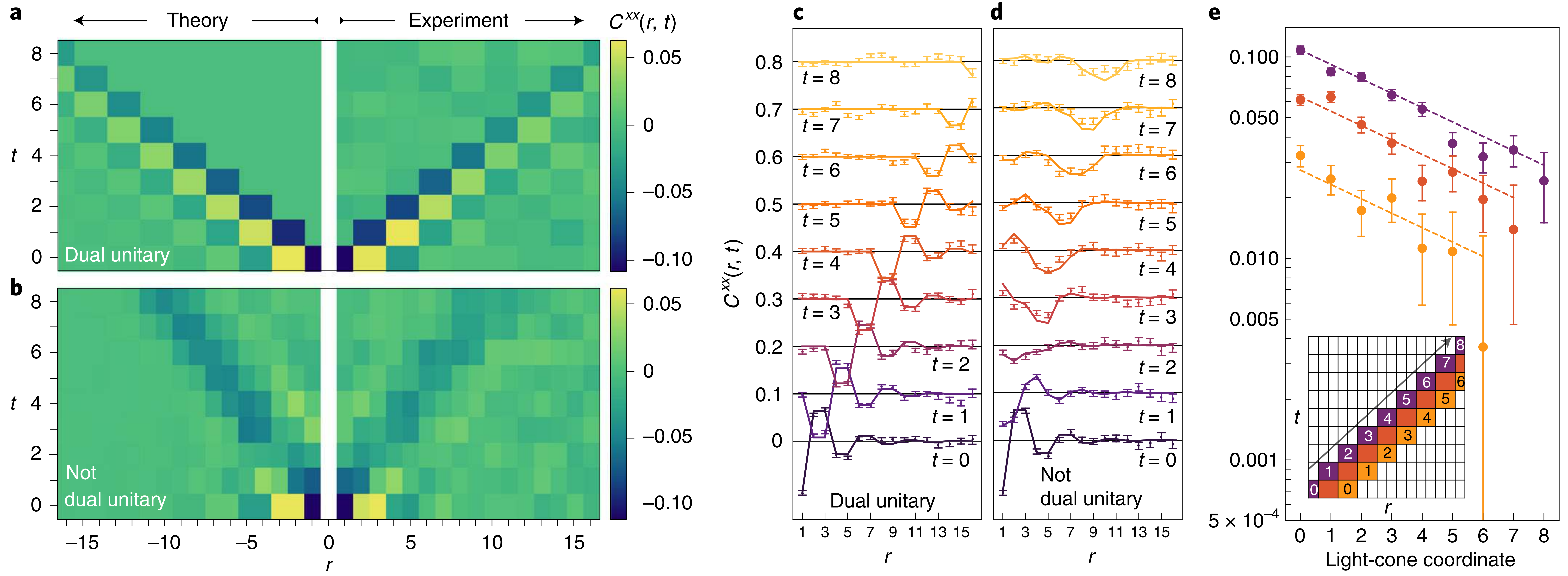


figure from [Claeys and Lamacraft, 2021]

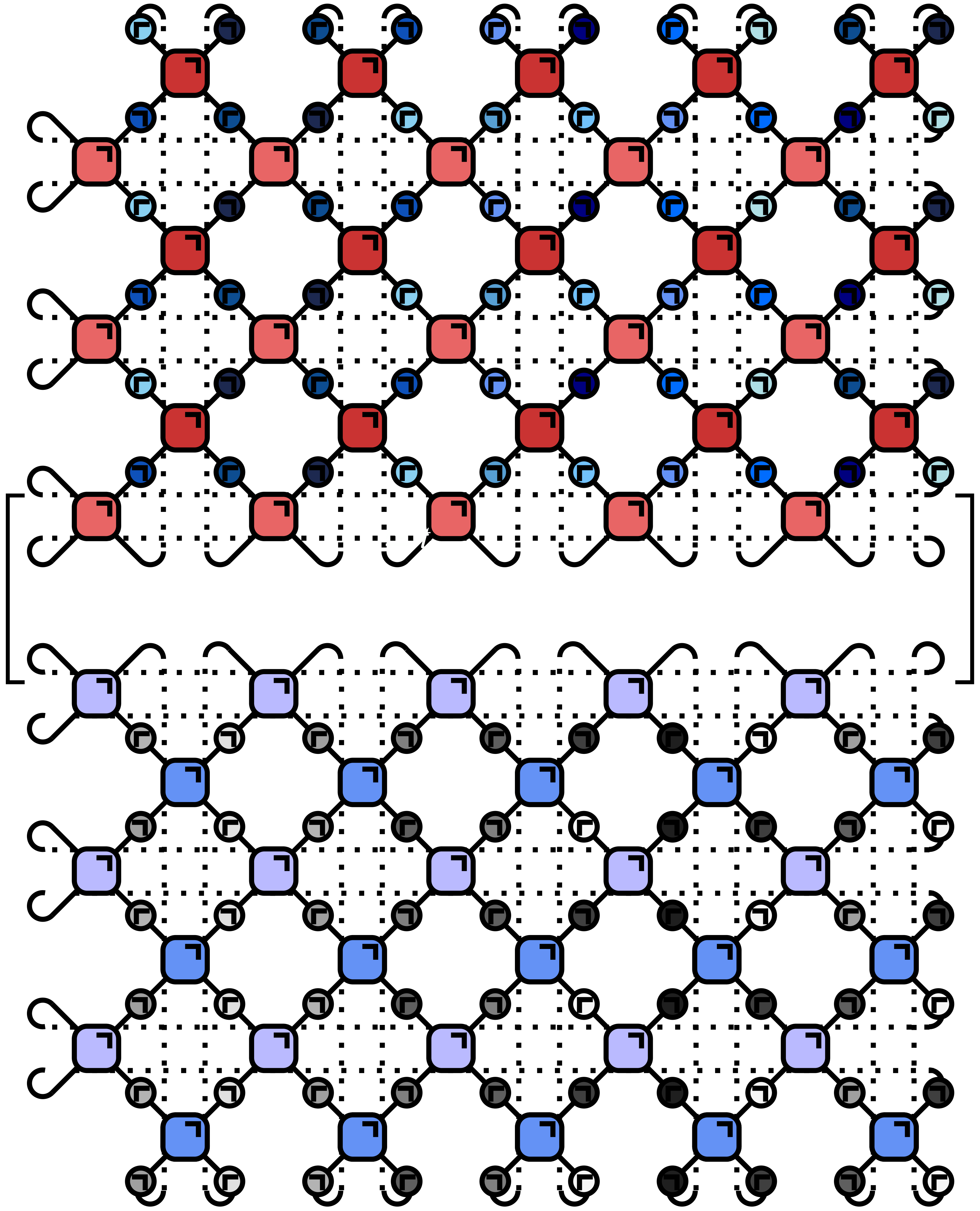
Experimental application of dual unitary circuits



[Chertkov et al., Nat. Phys. ’22]

Proof of quantum chaos conjecture for dual unitary circuits

$$K(t, L) = \mathbb{E}\left[\mathrm{tr}(\mathbb{U}_L)^t \mathrm{tr}(\mathbb{U}_L^\dagger)^t\right] = \mathbb{E}\left[\quad\right]$$



$$\simeq t$$

[Bertini, Kos, TP, 2018, 2021]

Conclusions

Despite many decades of efforts, there is still no unified view of quantum and classical chaos.

Yet, there are clear and universal paradigms of chaos in both worlds, with a few exactly solvable models characterizing them.